TechNote 206

Equations

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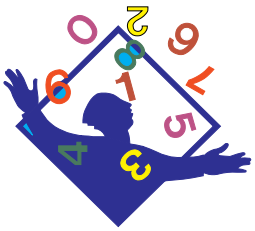

CONTENTS

I. INTRODUCTION

Values obtained through use of the following equations should be considered estimates, as the equations are based upon a number of theoretical assumptions. Values that are determined empirically or through use of analytical techniques are expected to differ to an extent. Some values (designated with an *) are provided on the Certificates of Analysis and order paperwork that accompanies microsphere shipments.

II. EQUATIONS

A. Solids Content*

$$S = \frac{m \bullet 100}{V_S} \quad \text{OR} \quad m = \frac{S \bullet V_S}{100}$$

where:
- S = weight/volume % solids*
- V_S = volume of suspension (mL)*
- m = mass of microspheres (g)*

(e.g., S = 10 for 5mL suspension, 0.5g microspheres)

B. % Coefficient of Variation (Size Distribution of the Microsphere Population)

$$\%\,CV = \frac{SD}{d} \times 100$$

where:
- % CV = % coefficient of variation (size distribution of the microsphere population)
- SD = standard deviation (µm)* (*Note*: Standard deviation is not provided for all products.)
- d = mean diameter (µm)*

C. # Microspheres/Gram*

$$N = \frac{6 \times 10^{12}}{\pi \bullet \rho_S \bullet d^3}$$

where:
- N = # microspheres/gram for dry powders*
- ρ_S = density of solid sphere (g/cm^3)*
- d = mean diameter (µm)*

D. # Microspheres/mL*

$$N = \frac{6 \times 10^{10} \bullet S \bullet \rho_L}{\pi \bullet \rho_S \bullet d^3}$$

where:
- N = # microspheres/mL for suspensions in water*
- S = weight % solids (for 10% solids suspension, S=10)*
- ρ_L = density of microsphere suspension (g/mL)*
- ρ_L = $100 \bullet \rho_S / [S\,(1-\rho_S) + (100 \bullet \rho_S)]$
- ρ_S = density of solid sphere (g/cm^3)*
- d = mean diameter (µm)*

E. Surface Area / Gram*

$$AG = \frac{6 \times 10^{12}}{\rho_S \bullet d}$$

where:
- AG = surface area/gram for dry powders (µm^2/g)*
- ρ_S = density of solid sphere (g/cm^3)*
- d = mean diameter (µm)*

F. Surface Area / mL*

$$A = \frac{6 \times 10^{10} \bullet S \bullet \rho_L}{\rho_S \bullet d}$$

where:
- A = surface area / mL for suspensions in water (µm^2/g)*
- S = weight % solids (for 10% solids solution, S=10)*
- ρ_L = density of microsphere suspension (g/mL)*
- ρ_L = $100 \bullet \rho_S / [S\,(1-\rho_S) + (100 \bullet \rho_S)]$
- ρ_S = density of solid sphere (g/cm^3)*
- d = mean diameter (µm)*

G. Parking Area (Surface Charge Density)*

$$P = \frac{1}{1.004 \bullet D_C \bullet \rho_S \bullet d}$$

where: P = parking area (Å^2/charge group)*
D_C = surface charge density or titration value (meq/g)* *(provided in µeq/g on COA)*
ρ_S = density of solid sphere (g/cm^3)*
d = mean diameter (µm)*

Notes:
a. Surface titer / parking area are not provided for all products.
b. For a detailed description of parking area, see "The Particle Doctor®" section of our March 2001 issue of *Painless Particles®*, which may be downloaded from our website, **www.bangslabs.com**.

H. # Charge Groups / Microsphere

$$C_M = \frac{\pi \bullet d^2 \bullet 10^8}{P}$$

where: C_M = # charge groups / microsphere
d = mean diameter (µm)*
P = parking area (Å^2)*

I. Settling Velocity (Specify Medium)

$$v = \frac{1}{18} d2 (\rho_S - \rho_0) \frac{10^{-6}}{h} G$$

where: v = settling velocity (cm / sec)
d = mean diameter (µm)*
ρ_S = density of solid sphere (g/cm^3)*
ρ_0 = medium density (g/cm^3)*
h = medium viscosity (poise = g/cm • sec)
G = gravity (980 cm/sec^2)

J. Settling Velocity (In Water)

$$V_m = 5.448 \times 10^{-5} \bullet (\rho_S - 1) \bullet d^2$$

where: V_m = maximum settling velocity (cm/sec) for a single microsphere settling in water at room temperature under the influence of normal gravitational force (1G)
ρ_S = density of solid sphere (g/cm^3)*
d = mean diameter (µm)*

K. Settling Velocity (In the Centrifuge)

$$V_h^{5\%} = \frac{2}{3} \bullet V_m \bullet G$$

where: $V_h^{5\%}$ = true settling velocity or hindered velocity (cm/sec) for a 5% w/w suspension of microspheres settling in water at room temperature
V_m = maximum settling velocity
G = multiples of earth gravitation constant, G forces

L. Settling Time

$$t = \frac{h}{V_h^{5\%}} \quad \text{OR} \quad \frac{h}{V_{ch}^{5\%}}$$

where: t = settling time (sec)
h = distance from the top of the liquid to the bottom layer of settled solids (cm)
$V_h^{5\%}$ = true settling velocity or hindered velocity (cm/sec) for a 5% w/w suspension of microspheres settling in water at room temperature under the influence of normal gravitational force (1G)
$V_{ch}^{5\%}$ = hindered settling velocity in the centrifuge (cm/sec) for a 5% w/w suspension of microspheres settling in water at room temperature

M. Surface Saturation (Protein)

$$S = \frac{6}{\rho_S d} \bullet C$$

where: S = amount of representative protein required to achieve surface saturation (mg protein/g microspheres)
ρ_S = density of solid sphere (g/cm^3)*
d = mean diameter (µm)*
C = capacity of microsphere surface for a given protein (mg protein/m^2 of sphere surface)

Notes:
a. C ~ 3 mg/m^2 for BSA [MW 65kD],
C ~ 2.5 mg/m^2 for bovine IgG [MW 150kd].[1]
b. See Tech Note 204, *Adsorption to Microspheres*, and TechNote 205, *Covalent Coupling*, for more detailed information.

III. SAMPLE VALUES

Table 1: Sample Values

Diameter (Microns)	Beads per gram	Beads per mL	Surface Area (µm^2/g)	Surface Area (µm^2/mL)	Settling Velocity (cm/sec)
0.052	$1.3x10^{16}$	$1.3x10^{15}$	$1.1x10^{14}$	$1.1x10^{13}$	$7.4x10^{-9}$
0.100	$1.8x10^{15}$	$1.8x10^{14}$	$5.7x10^{13}$	$5.7x10^{12}$	$2.7x10^{-8}$
0.500	$1.5x10^{13}$	$1.5x10^{12}$	$1.1x10^{13}$	$1.1x10^{12}$	$6.8x10^{-7}$
1.000	$1.8x10^{12}$	$1.8x10^{11}$	$5.7x10^{12}$	$5.7x10^{11}$	$2.7x10^{-6}$
2.500	$1.2x10^{11}$	$1.1x10^{10}$	$2.3x10^{12}$	$2.3x10^{11}$	$1.7x10^{-5}$
10.00	$1.8x10^{9}$	$1.8x10^{8}$	$5.7x10^{11}$	$5.7x10^{10}$	$2.7x10^{-4}$
25.00	$1.2x10^{8}$	$1.2x10^{7}$	$2.3x10^{11}$	$2.3x10^{10}$	$2.0x10^{-3}$
108.0	$1.4x10^{6}$	---	$5.3x10^{10}$	---	---
500.0	$1.4x10^{4}$	---	$1.1x10^{10}$	---	---

Notes: a. Calculations for 0.052-25.0µm are based on a suspension of polystyrene microspheres (density = 1.05 g/cm^3) at 10% solids (w/v).
b. 108 and 500µm diameter microsphere calculations compositions are based on compositions of poly(styrene/2% divinylbenzene), density = 1.06 g/cm^3, and the calculations are based on dry presentation.

IV. REFERENCES

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2. **Bangs, L.B.** 1987. *Uniform latex particles.* Indianapolis: Seragen Diagnostics, Inc.